V E R I F I C A T I O N # 2

# Sliding

The purpose of this verification is to confirm that the sliding algorithm used by the program is working correctly. The sliding algorithm is used to calculate the motion of the rocks while they are in contact with the slope surface. The sliding algorithm is executed every time the rock stops moving. This stopping occurs, and thus the sliding algorithm is executed, at least once every simulation. Along with the projectile algorithm, the sliding algorithm determines the position and velocity of the rock. Any errors in the sliding algorithm would produce erroneous locations and velocities in the output. Therefore, it is essential that the sliding algorithm work correctly.

The sliding verification consists of four very similar examples that were designed to test every potential situation that the sliding algorithm might encounter. No statistics were used in this verification (i.e. only mean values were used, all standard deviations were set to 0).

In order to guard against numerical instability; at the beginning of each simulation the program offsets the rock *slightly* into the analysis area (i.e. slightly into the “air”) and allows it to fall under the influence of gravity. The offset generated by the program, for the geometry used in this verification, was less than 0.02 mm. This “offsetting” requires the program to execute the projectile algorithm, at least once, before entering the sliding algorithm. This causes the program's results to differ slightly when compared to the manual calculations.

It should be noted that this “offsetting” will have a minimal effect on the outcome of a typical simulation. Since the offset is usually very small (0.02 mm in this case) and it is only applied once, at the beginning of the simulation, it will have a negligible effect on most simulations.

## Initial Conditions

The slope geometry and coefficients of restitution used for all four cases were identical. The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

| | x co-ordinate | y co-ordinate | $R_N$ | $R_T$ |
|---|---|---|---|---|
| Vertex 1 | 0 | 1 | | |
| Segment 1 | | | 0 | 1 |
| Vertex 2 | 1 | 1 | | |
| Segment 2 | | | 0 | 1 |
| Vertex 3 | 3 | 5 | | |
| Segment 3 | | | 0 | 1 |
| Vertex 4 | 8 | 6.5 | | |
| Segment 4 | | | 0 | 1 |
| Vertex 5 | 12 | 0 | | |

*Table A.2.1 - Slope geometry*

The coefficient of tangential restitution ($R_T$) was set to 1 and the coefficient of normal restitution ($R_N$) was set to 0 in all cases. The choice of $R_N = 0$ was made in order to force the program to initiate the sliding algorithm, immediately after the first pass through the projectile algorithm. The choice of $R_T = 1$ was made in order to minimise the change in tangential velocity during the pass through the projectile algorithm.

The rocks were started at $X_0 = 6.5$ m, $Y_0 = 6.05$ m (which lies on segment 3) in all four cases. Because the rocks were placed directly on segment 3 and were given a velocity that was tangential to the surface, all sliding occurred on segment 3.

It will be useful to note that the slope of the segment on which sliding occurs (segment 3) is:

$$\theta = \tan^{-1}\left(\frac{6.5-5}{8-3}\right) \cong 16.7^0$$

The initial velocity of the rocks and the friction angle of the slope are the only parameters that changed, depending on the case being considered. The difference between the four cases are summarised in a table:

| Case | $V_{X0}$ | $V_{Y0}$ | $\phi$ | Description |
|---|---|---|---|---|
| 1 | -1.0 | -0.3 | 10° | Sliding downhill and off of the segment |
| 2 | -1.0 | -0.3 | 18° | Sliding downhill and stopping |
| 3 | 3.7 | 1.11 | 10° | Sliding uphill and off end of segment |
| 4 | 3.7 | 1.11 | 18° | Sliding uphill and stopping |

*Table A.2.2 - Difference between cases*

### Case 1: Sliding downhill and off of the segment

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the downslope direction, and the conditions were such that the rock would slide off the downslope end of the segment. The rock was given an initial velocity of $V_{X0}$ = -1 m/s, $V_{Y0}$ = -0.3 m/s. The friction angle of the slope was set at 10°. Since the slope angle is greater than the friction angle the rock will slide off of the end of the segment. The exit velocity was calculated (using equation 4.19):

$$V_{EXIT} = \sqrt{V_0^2 - 2sgk} = \sqrt{(1.044)^2 - 2(3.654)(-9.81)(0.1185)} \cong 3.095\ m/s$$

where:

$$V_0 = \sqrt{V_{X0}^2 + V_{Y0}^2} = \sqrt{(-1)^2 + (-0.3)^2} \cong 1.044\ m/s$$

$$s = \sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2} = \sqrt{(3-6.5)^2 + (5-6.05)^2} \cong 3.654\ m$$

$$k = \sin(\theta) - \cos(\theta)\tan(\phi) = \sin(16.67^0) - \cos(16.67^0)\tan(10^0) = 0.1185$$

In order to check the program's results, a “data collector” was added in RocFall. The data collector was added from co-ordinates (3, 4.5) to co-ordinates (3, 5.5). The results were graphed using the “graph impact velocity” option in RocFall. The exit velocity was obtained from the graph using the “copy raw data” option.

The results are presented in the following table:

| | Hand Calculation | RocFall | Difference |
|---|---|---|---|
| Exit Velocity | 3.095 | 3.098 | 0.1% |

*Table A.2.3 - Sliding downhill and off of the segment, comparison of results*

The results are very similar. The reason for the difference is that, in the program the rock gains some velocity falling from its offset position and starts with a velocity *slightly* greater than the value that was used in the hand calculations.

## Case 2: Sliding downhill and stopping

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the downslope direction, and the conditions were such that the rock would be slowed by friction and stop before reaching the downslope end of the segment. The rock was given an initial velocity of $V_X$ = -1 m/s, $V_Y$ = -0.3 m/s. The friction angle of the slope was set at 18°. Since the angle of the slope is less than the friction angle the rock will slow down, and depending on the length of the segment, stop before reaching the end of the segment. A calculation is made to see how far the rock will slide before stopping (using equation 4.20):

$$s = \frac{V_0^2}{2gk} = \frac{(1.044)^2}{2(-9.81)(-0.0239)} = 2.328\ m$$

where:

$$V_0 = \sqrt{V_{X0}^2 + V_{Y0}^2} = \sqrt{(-1)^2 + (-0.3)^2} \cong 1.044\ m/s$$

$$k = \sin(\theta) - \cos(\theta)\tan(\phi) = \sin(16.67^0) - \cos(16.67^0)\tan(18^0) = -0.0239$$

The rock will stop moving at a point 2.328 m downslope of where it began sliding. A check is made to see if the rock will reach the end of the segment before it stops sliding. The distance to the end of the segment is calculated:

$$s_D = \sqrt{(X_0 - X_1)^2 + (Y_0 - Y_1)^2} = \sqrt{(6.5-3)^2 + (6.05-5)^2} \cong 3.65\ m$$

Since the stopping distance, $s$ (= 2.328 m) is less than the distance to the end of the segment, $s_D$ (= 3.65 m) the rock will stop before sliding off of the end of the segment. The location where the rock stopped is calculated:

$$x = X_0 - (s)\cos(\theta) = 6.5 - (2.328)\cos(16.67^0) = 4.27\ m$$

$$y = Y_0 - (s)\sin(\theta) = 6.05 - (2.328)\sin(16.67^0) = 5.38\ m$$

In order to check the program's results the simulation was performed in RocFall. The results were graphed using the "graph rock endpoints" option in RocFall. The location of the rock endpoint was obtained from the graph using the "copy raw data" option. The results are presented in the following table:

| | Hand Calculation | RocFall | Difference |
|---|---|---|---|
| x co-ordinate of endpoint | 4.270 | 4.207 | 1.5% |
| y co-ordinate of endpoint | 5.381 | 5.362 | 0.4% |

*Table A.2.4 - Sliding downhill and stopping , comparison of results*

The results are very similar. The reason for the difference was found by stepping through the program as it executed. All values were identical to the manual calculations except for the value of $V_0$. The value of $V_0$ in the program was 1.118 m/s. The value calculated by hand was 1.044 m/s. The higher value of $V_0$ in the program was caused by the rock being offset into the analysis area at the beginning of the simulation and then gaining some velocity by falling under the influence of gravity.

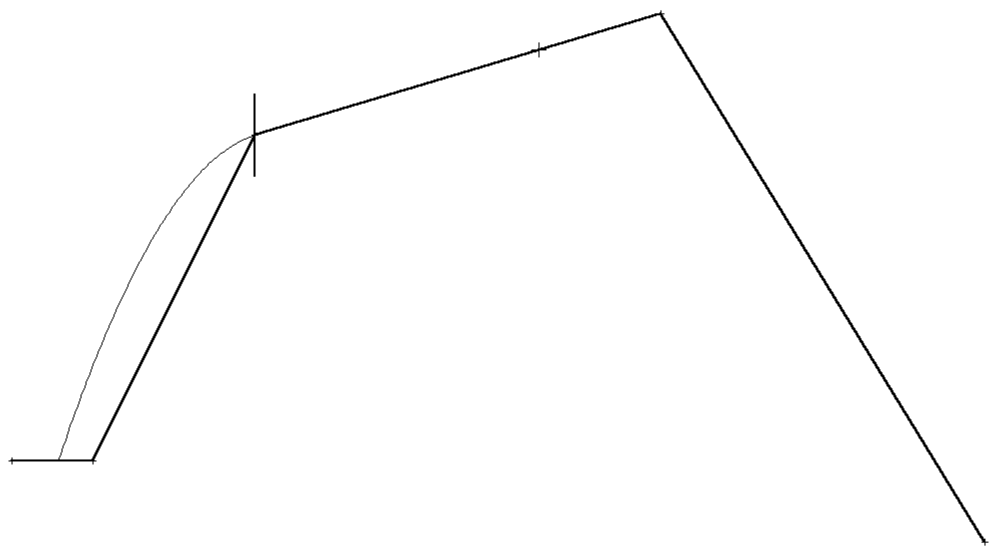

*Figure A.2.1 - Sliding downhill and off of the segment*

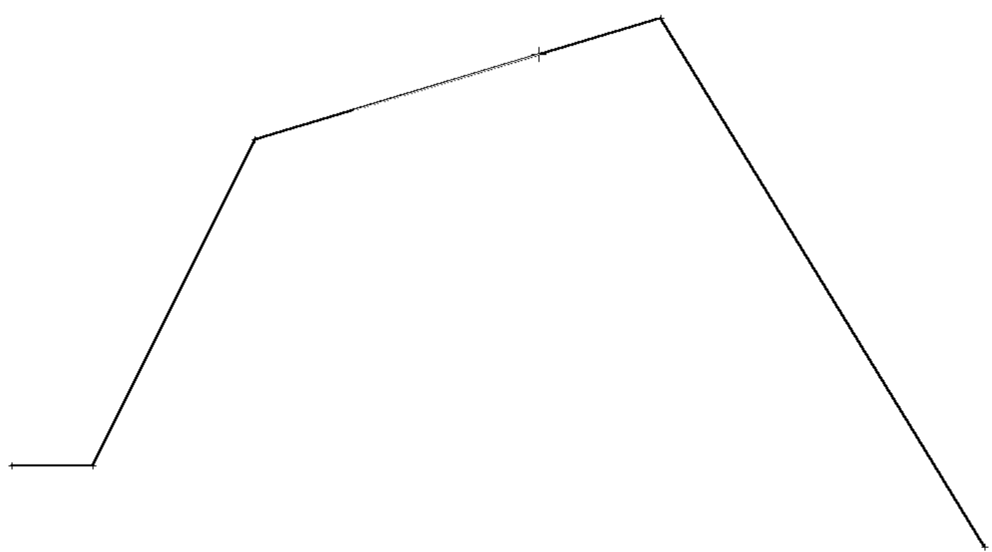

*Figure A.2.2 - Sliding downhill and stopping*

### Case 3: Sliding uphill and off of the segment

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the upslope direction and the conditions were such that the rock would slide off the upslope end of the segment. The rock was given an initial velocity of $V_X$ = 3.7 m/s, $V_Y$ = 1.11 m/s. The friction angle of the slope was set at 10°. The exit velocity is calculated:

$$V_{EXIT} = \sqrt{V_0^2 - 2sgk} = \sqrt{(3.863)^2 - 2(1.566)(-9.81)(-0.4562)} \cong 0.953\ m/s$$

where:

$$V_0 = \sqrt{(3.7)^2 + (1.11)^2} \cong 3.863\ m/s$$

$$s = \sqrt{(X_2 - X_0)^2 + (Y_2 - Y_0)^2} = \sqrt{(8-6.5)^2 + (6.5-6.05)^2} \cong 1.566\ m$$

$$k = -\sin(\theta) - \cos(\theta)\tan(\phi) = -\sin(16.67^0) - \cos(16.67^0)\tan(10^0) = -0.4562$$

In order to check the results from the program a "data collector" was added in RocFall. The data collector was added from co-ordinates (8, 6) to co-ordinates (8, 7). The results were graphed using the "graph impact velocity" option in RocFall. The exit velocity was obtained from the graphs using the "copy raw data" option in the program. The results are presented in the following table:

| | Hand Calculation | RocFall | Difference |
|---|---|---|---|
| Exit Velocity | 0.953 | 0.984 | 3.1% |

*Table A.2.5 - Sliding uphill and off the segment, comparison of results*

The results are similar. The reason for the difference was found by stepping through the program as it executed. All of the values in the program were identical to the manually calculated values except for $V_0$ and $s$. The value of $s$ in the program was 1.548 m (vs. 1.566 m for the hand calculation). The value of $V_0$ in the program was 3.849 m/s (vs. 3.863 m/s for the hand calculation). The lower values of $V_0$ and $s$ in the program were caused by the rock being offset into the analysis area and then gaining some velocity and changing position during the fall from the offset position.

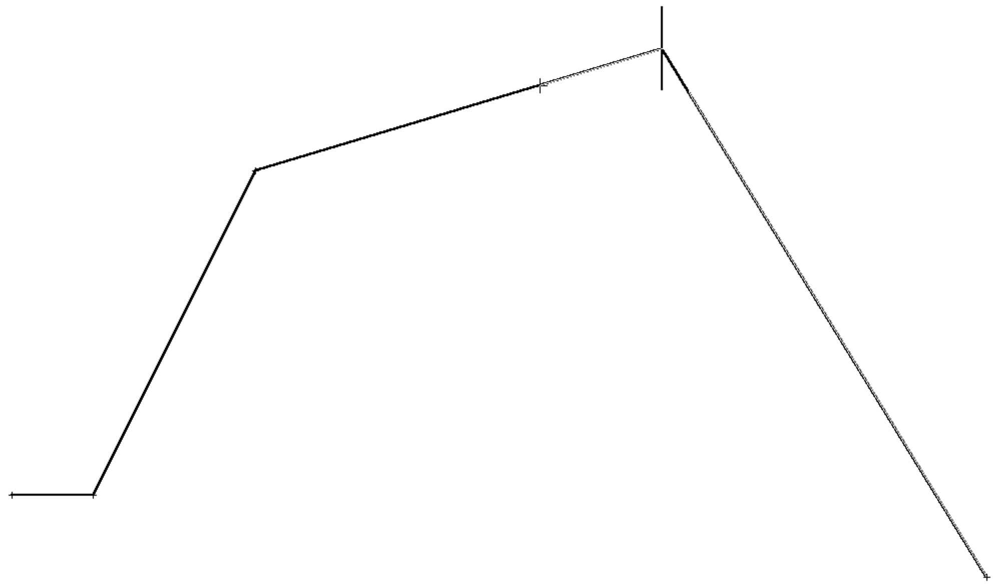

*Figure A.2.3 - Sliding uphill and off of the segment*

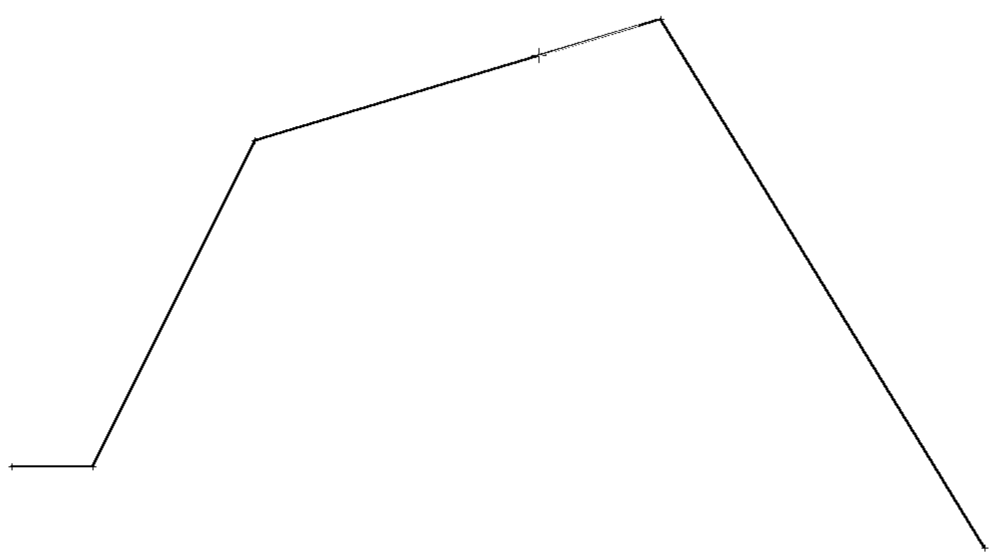

*Figure A.2.4 - Sliding uphill and stopping*

### Case 4: Sliding uphill and stopping

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the upslope direction, and the conditions were such that the rock would slow down and stop before it reached the upslope end of the segment. The rock was given an initial velocity of $V_X$ = 3.7 m/s, $V_Y$ = 1.11 m/s. The friction angle of the slope was set at 18°. Since the angle of the slope is less than the friction angle the rock will slow down, and depending on the length of the segment, stop before reaching the end of the segment. A calculation is made to see how far the rock will slide before stopping:

$$s = \frac{V_0^2}{2gk} = \frac{3.863^2}{2(-9.80665)(-0.599)} = 1.271\, m$$

where:

$$V_0 = \sqrt{(3.7)^2 + (1.11)^2} \cong 3.863\ m/s$$

$$k = -\sin(\theta) - \cos(\theta)\tan(\phi) = -\sin(16.67^0) - \cos(16.67^0)\tan(18^0) = -0.599$$

The rock will stop moving at a point 1.271 m upslope of where it began sliding. A check is made to see if the rock with reach the end of the segment before it stops sliding. The distance to the end of the segment is calculated:

$$s_D = \sqrt{(X_0 - X_2)^2 + (Y_0 - Y_2)^2} = \sqrt{(6.5-8)^2 + (6.05-6.5)^2} \cong 1.570\ m$$

Since the stopping distance, $s$ (= 1.271 m) is less than the distance to the end of the segment, $s_D$ (= 1.570 m) the rock will stop before reaching the end of the segment. The location where the rocks stops is calculated:

$$x = X_0 + (s)\cos(\theta) = 6.5 + (1.271)\cos(16.67^0) = 7.717\ m$$

$$y = Y_0 - (s)\sin(\theta) = 6.05 + (1.271)\sin(16.67^0) = 6.415\ m$$

In order to check the program's results the simulation was performed in RocFall. The results were graphed using the "graph rock endpoints" option in RocFall. The location of the rock endpoint was obtained from the graph using the "copy raw data" option. The results are presented in the following table:

| | Hand Calculation | RocFall | Difference |
|---|---|---|---|
| x co-ordinate of endpoint | 7.717 | 7.727 | 1.3% |
| y co-ordinate of endpoint | 6.415 | 6.418 | 0.05% |

*Table A.2.6 - Sliding uphill and stopping, comparison of results*

The results are very similar. The reason for the difference was found by stepping through the program as it executed. All of the values in the program were identical to the manual calculations except for the value of $V_0$. The value of $V_0$ in the program was 3.850 m/s (vs. 3.863 m/s for hand calculation). This was caused by the rock being "offset into the analysis area" and then gaining some velocity by falling under the influence of gravity.

## Conclusion

There is one additional case that has not been can be presented: the case where the rock slides uphill, stops, and then slides back down and off the downslope end of the segment. This was not presented as a separate case because it is dealt with in the program by treating it as two separate cases that have been considered (case 4 followed by case 1).

The sliding algorithm seems to be working correctly. In each of the four cases the difference between the manual calculations and the results produced by RocFall were explained by the "offsetting", and not by errors in the sliding algorithm.

The results are presented as they are (with the slight difference caused by the offsetting), so that anyone using RocFall could duplicate the verification cases. Although the results from the manual calculations could have been duplicated exactly (by inserting the values directly into the sliding algorithm) this was not done, because this option is only available to the program developer and not to someone using RocFall.

Considering how poorly defined many of the significant quantities (such as $R_T$) are, the consequences of moving the initial rock position a few hundredths of a millimetre at the beginning of each simulation can be ignored in the majority of simulations.